Rope Dynamics

by Phil Andrew, MSc, MPhil and Stefan Kaczmarczyk, PhD

Learning Objectives

After reading this article, you should have learned about:

- How transient vibrations arise
- Why rope vibrations often occur at particular positions in the hoistway
- Why, when considering transient rope vibration, even a high-speed elevator may be treated as if it were stationary
- Why lateral rope vibrations may give rise to longitudinal vibration and vice versa
- Why transient rope vibration may be very difficult to eliminate

Rope vibrations in an elevator system usually result in a ride quality that is unacceptable. A well-known phenomenon is "transient" vibration, which occurs at a specific point in the elevator travel, usually near the upper floors, and can be extremely difficult to eliminate. We will seek to show that oscillations in the roping system may be considered "rapid" in comparison with the rated speed of the elevator (even for "high-speed" elevators rated at 12 mps [2400 fpm] or more). We will show how, for the purpose of analyzing the dynamic behavior of the roping system, a moving elevator may be considered "quasi-stationary."

Hamilton's Principle and classical mechanics will be employed to derive the dynamic equations that describe the oscillation of the ropes. The resulting partial differential equations will be presented in order to explain how lateral and longitudinal oscillations in the suspension system turn out to be cross coupled, so that a lateral oscillation of the ropes can initiate longitudinal vibrations and vice versa. Finally, we will work through an example to show how transient vibrations may arise at particular locations in the elevator travel and highlight why such rope oscillation and vibration is so difficult to eliminate, and look briefly at some of the solutions to the problem that have been proposed in the past.

Suspension System Dynamics

There are likely very few elevator engineers who have not been faced with an elevator system exhibiting unwanted vibration in one form or another. The underlying causes of vibration in an elevator system are varied, including poorly aligned guiderail joints, eccentric pulleys and sheaves, systematic resonance in the electronic control system, and gear and motor generated vibrations.

In many cases, an elevator will not vibrate throughout its travel, but will "pass through" a resonant vibration at some particular stage in the travel. Very often, this vibration stage occurs at or near the highest floor, as the suspension ropes become short. Figure 1 indicates the kind of phenomenon that might be experienced.

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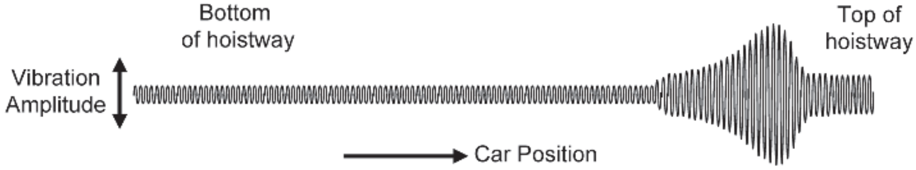

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However, whatever the underlying cause of the vibration, in almost all cases, the vibration will excite an associated vibration in the ropes, whether it be the suspension ropes or the compensation ropes. The vibration is thus coupled into the car with consequent deterioration in ride quality. The term "vibration" has colloquial connotations of relatively high frequency and may be confusing if we intend to include "rope sway" in the discussion. Consequently, we shall employ the term "oscillation" in preference to "vibration," since this term has a colloquial context of a wider frequency range.

Let us start with a simple model. Consider the suspension and compensation ropes of a stationary elevator. As we have discussed, the suspension will be stretched elastically by the mass of the elevator and its load. In the vertical direction, the elevator car is free to move and can oscillate on the "spring" of the suspension ropes as shown in Figure 2(a). We shall designate oscillations in the vertical direction by the variable u, indicating vertical displacement from the quiescent position, $\dot{u}$ indicating the vertical velocity of the oscillation of the elevator car (i.e., not its velocity of travel through the hoistway) and $\ddot{u}$ indicating vertical acceleration associated with the oscillation.

Since the elevator car is held in the guides and cannot move freely in the horizontal direction, lateral oscillations of the suspension ropes are constrained at each end. The oscillations are constrained in a similar (though not quite identical) manner to a guitar or violin string (Figure 2(b) and (c)). Nevertheless, the rope can oscillate in any horizontal direction. In order to generalize the discussion as far as possible, we will resolve the lateral oscillations into orthogonal displacements, v, $\dot{v}$ and $\ddot{v}$ indicating motion in the plane of the guides (in plane), and w, $\dot{w}$ and $\ddot{w}$ orthogonal to the plane of the guides (out of plane).

Thus, if the ropes oscillate in a direction x at some angle θ to the plane of the guides, then the in-plane and out-of-plane motions will be

$v = x\cos\theta$, $\dot{v} = \dot{x}\cos\theta$ and $\ddot{v} = \ddot{x}\cos\theta$

in the plane of the guides, and

$w = x\sin\theta$, $\dot{w} = \dot{x}\sin\theta$ and $\ddot{w} = \ddot{x}\sin\theta$

in the orthogonal plane.

Of course, we might have a situation where the ropes are "whirling" in their oscillations; in which case the angle θ will itself be a function of time, i.e., $\theta = \theta(t)$.

The nature of current suspension-rope materials is such that the damping of any rope vibrations is fairly small.

An analysis of rope oscillation based on elementary mechanics might indicate that if M is the total suspended mass (kg) (not including the mass of the suspension ropes) and k is the stiffness of the ropes (N/m), then in the vertical plane, we would have a potential "natural" or "resonant" oscillation frequency:

$$\omega_{u0} = \sqrt{\frac{k}{M}} \text{ rad/s} \quad \text{(Equation 1)}$$

Even if we did allow for the mass of the suspension ropes, the simple model above does not reflect the true situation. In practice, there will be additional harmonic frequencies that can lead to phenomena apparent to the passenger.

In the lateral direction, we might expect oscillation frequencies

$$\omega_{xn} = n\frac{\pi}{L}\sqrt{\frac{\text{Rope Tension}}{\text{Rope Mass}}} = n\frac{\pi}{L}\sqrt{\frac{Mg_n}{n_{SR}m_{SR}}} \text{ rad/s}, \quad n = 1, 2, 3, 4....$$

where L is the length of suspension rope, n_{SR} is the number of suspension ropes and m_{SR} is the mass/m of the suspension rope, noting once more that Mg_n, the equation for rope tension, does not include the mass of the suspension rope itself. With lateral oscillation, even a simple analysis indicates that harmonic frequencies are possible.

Although this gives us an intuitive idea of what we might expect, the situation is significantly more complex. In order to get a more realistic picture of the oscillation modes of the suspension ropes, we will have to take into account more complex phenomena than we have considered so far.

The "Slowly Varying" System

The first complicating factor is the issue raised by the motion of the elevator car itself. Our simplistic picture of the oscillation mechanisms has been based on a stationary elevator car; in practice, the elevator may be in motion, so that the length of the suspension ropes is changing with time. The suspended mass $M(t)$ and rope length $L(t)$ are now functions of time. The suspended mass $M(t)$, which still does not account for the mass of the suspension ropes, will vary due to the changing mass of compensation ropes/chains and traveling cables suspended from the car as the elevator travels through the hoistway. Even in our simplistic model, the stiffness k will vary as the length of the ropes changes, varying the longitudinal frequency ω_{u0}, and, of course, the change in length directly affects the lateral frequencies ω_{xn}.

Kaczmarczyk[1] has shown that we can define a dimensionless parameter

$$\varepsilon = \frac{V/L(t)}{\omega_0} = \frac{V}{\omega_0 L(t)} \quad \text{(Equation 2)}$$

where V is the elevator rated speed (mps), ω_0 rad/s is the lowest natural frequency (either lateral or longitudinal), and $L(t)$ is the suspension rope length (m).[1] Note that the issue here is not the speed of the ropes, but the rate at which the ropes are shortening. Consequently, independent of the reeving (1:1, 2:1, etc.) the parameter ε is a function of elevator speed, not rope speed.

Kaczmarczyk (ibid.) reports that if we can be satisfied that

$$\varepsilon \ll 1 \quad \text{(Equation 3)}$$

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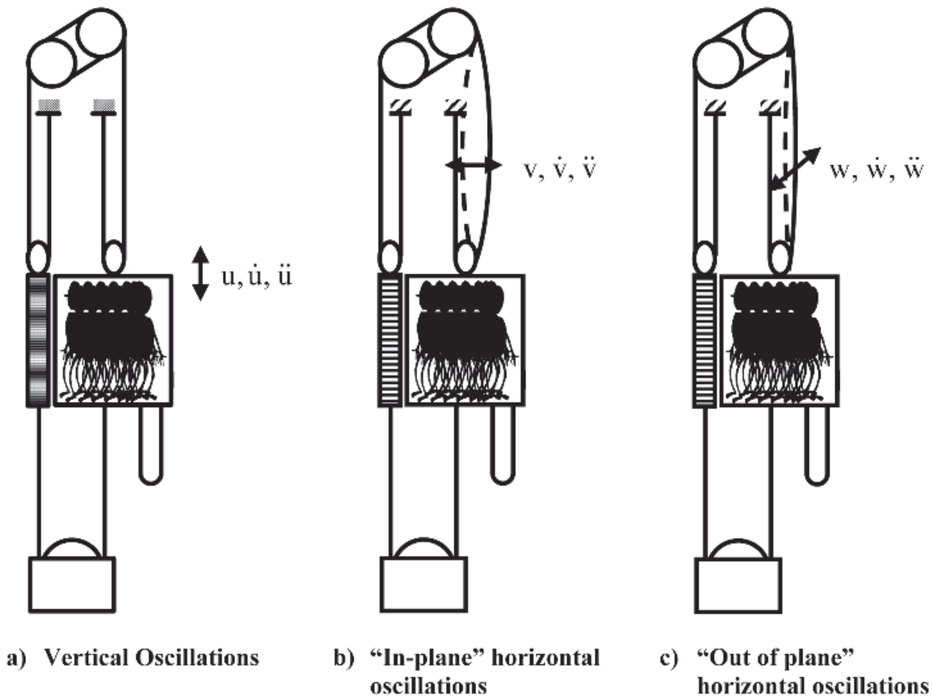

Figure 2: Vibration modes

then we can define the system as varying slowly, meaning that we can take any particular position of the elevator, whether or not it is in motion, and treat it as if the elevator were stationary.[1]

Looking at lateral oscillations of the ropes, the elementary treatment referred to above suggests that the natural frequency of the ropes will be

$$\omega_{x0} = \frac{\pi}{L(t)}\sqrt{\frac{\text{Rope Tension}}{\text{Rope Mass}/m}} = \frac{\pi}{L(t)}\sqrt{\frac{M(t)g_n}{n_{SR}m_{SR}}} \qquad \text{(Equation 4)}$$

However, this simple equation assumes that the rope itself has zero bending stiffness and is orientated horizontally. Sun[2] suggests that for a more accurate estimate of the mean natural frequency of a vertical suspension, we need to account for the influence of the rope mass on the mean tension. The mean rope tension *T* needs to include half the weight of the rope, i.e.

$$\text{Mean Rope Tension} = M(t)g_n + \frac{L(t)}{2}n_{SR}m_{SR}g_n$$

Including lateral harmonic oscillation frequencies as well as the natural frequency, Equation 4 then becomes

$$\overline{\omega_{xn}} = \frac{n\pi}{L(t)}\sqrt{\frac{\text{Mean Rope Tension}}{\text{Rope Mass/m}}} = \frac{n\pi}{L(t)}\sqrt{\frac{M(t)g_n + \frac{L(t)}{2}n_{SR}m_{SR}g_n}{n_{SR}m_{SR}}}$$

i.e. $$\overline{\omega_{xn}} = \frac{n\pi}{L(t)}\sqrt{\frac{M(t)g_n}{n_{SR}m_{SR}} + \frac{L(t)g_n}{2}} \qquad n = 1, 2, 3 \qquad \text{(Equation 5)}$$

The term $\frac{L(t)g_n}{2}$ inside the square-root sign suggests that the natural frequency of the oscillation will be higher than is suggested by the simple analysis of a horizontal, stretched string, and that with longer ropes (i.e., longer travel), the natural frequency of the ropes will not fall off as rapidly as

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would be predicted by the simple model of Equation 4. Nevertheless, the frequency will still be at a minimum when the car is at the lowest point of travel, leading to the largest value for ε. If we define L_{max} as the rope length when the car is at the lowest point in its normal travel, then

$$\overline{\omega_{xn}}\Big|_{min} = \frac{n\pi}{L_{max}}\sqrt{\frac{M(t)g_n}{n_{SR}m_{SR}} + \frac{L_{max}g_n}{2}}$$

Given that the loading on the ropes is governed by safety standards, we can relate Equation 5 to the rope characteristics via the guaranteed minimum breaking load of the rope and the safety factor.

We will define S_{fM} as the factor of safety at the lowest point of travel (varying, mainly depending on passenger load but also due to changes in total car side suspended mass as the elevator travels through the hoistway), and F_{min} as the specified minimum breaking load for the rope. With these definitions, then at the lowest point of travel

$$\left[\frac{M(t)}{n_{SR}} + m_{SR}L_{max}\right]g_n = \frac{F_{min}}{S_{fM}}$$

$$\text{i.e.}\quad \frac{M(t)g_n}{m_{SR}n_{SR}} = \frac{F_{min}}{S_{fM}m_{SR}} - L_{max}g_n \quad \text{...........(Equation 6)}$$

Combining Equations 5 and 6, the lowest mean natural frequency can be expressed as

$$\overline{\omega_{xn}}\Big|_{min} = \frac{n\pi}{L_{max}}\sqrt{\frac{M(t)g_n}{n_{SR}m_{SR}} + \frac{L_{max}g_n}{2}}$$

$$= \frac{n\pi}{L_{max}}\sqrt{\left(\frac{F_{min}}{S_{fM}m_{SR}} - L_{max}g_n\right) + \frac{L_{max}g_n}{2}}$$

$$= \frac{n\pi}{L_{max}}\sqrt{\frac{F_{min}}{S_{fM}m_{SR}} - \frac{L_{max}g_n}{2}} \quad \text{.............(Equation 7)}$$

Combining Equations 2 and 7, and given that, in practice, the lateral oscillation frequencies will be lower than the longitudinal frequency, the largest value of ε will be

$$\varepsilon_{max} = \frac{V}{\overline{\omega_{x0}}\Big|_{min} L_{max}} = \frac{V}{L_{max} \times \frac{\pi}{L_{max}}\sqrt{\frac{F_{min}}{S_{fM}m_{SR}} - \frac{L_{max}g_n}{2}}}$$

$$= \frac{V}{\pi\sqrt{\frac{F_{min}}{S_{fM}m_{SR}} - \frac{L_{max}g_n}{2}}} \quad \text{................(Equation 8)}$$

Taking the rope tables for a range of standard, fiber-cored suspension ropes, we can examine the value of $\frac{F_{min}}{m_{SR}}$ for various rope sizes and safety factors. Unsurprisingly, Table 1 shows that for any given value of safety factor S_f, the value of $\frac{F_{min}}{m_{SR}}$ does not vary significantly in the range of standard rope sizes between 11 mm and 19 mm.

Clearly, the material tensile stress at the minimum breaking load will be the same, and for similar rope construction, the rope space factor (steel area:total area) will also be reasonably constant, leading to the steady value for $\frac{F_{min}}{m_{SR}}$.

	Rope Diameter	mm	11	13	16	19
Fibre Core	Mass/m	kg/m	0.416	0.582	0.872	1.228
	Min Breaking load	kN	58.6	81.9	122.5	172.1
	$\frac{F_{min}}{m_{SR}}\times 10^{-5}$	m^2/s^2	1.409	1.407	1.405	1.401
	i.e. $\frac{F_{min}}{m_{SR}}\times 10^{-5} = 1.405 \pm 0.28\%$					
Steel Core	Mass/m	kg/m	0.500	0.710	1.070	1.490
	Min Breaking load	kN	77.4	108.9	163.6	228.5
	$\frac{F_{min}}{m_{SR}}\times 10^{-5}$	m^2/s^2	1.548	1.534	1.529	1.536
	i.e. $\frac{F_{min}}{m_{SR}}\times 10^{-5} = 1.539 \pm 0.62\%$					

Table 1: Values of $\frac{F_{min}}{m_{SR}}$ for standard suspension rope

With significantly smaller rope sizes, as are applied in some special designs, the space factor of the rope will begin to change, modifying the calculated value of $\frac{F_{min}}{m_{SR}}$. Of course, during system design, the minimum permissible safety factor will be determined for the case where the elevator car is carrying rated load, taking into account both standards requirements and the constraints of achieving a satisfactory rope life.

Any other loading is accounted for by calculating the higher safety factor associated with the lower loading, e.g., if the rated load is 1200 kg and the car-side fixed mass is 1600 kg, then if the system is designed for a safety factor of 16 at rated load, the safety factor with empty car will be

$$S_f\big|_{NL} = S_f\big|_{FL} \times \frac{2800}{1600} = 16 \times \frac{7}{4} = 28$$

If, as a conservative estimate, we base our analysis on an empty car with maximum factor of safety value 30, then Equation 8 becomes

$$\varepsilon_{max} = \frac{V}{\pi\sqrt{\frac{1.405\times 10^5}{30} - \frac{9.81}{2}L_{max}}} = \frac{V}{\pi\sqrt{4683 - 4.905L_{max}}} \quad \text{...(Equation 9)}$$

We can now investigate ε_{max} for a range of rated speed and travel. For this purpose, we will take the maximum practical travel as the lesser of 300 m or the travel distance the elevator can complete in a maximum of 60 s, allowing for constant acceleration/deceleration profiles at 1 mps^2 (i.e., ignoring limitations imposed by jerk requirements).

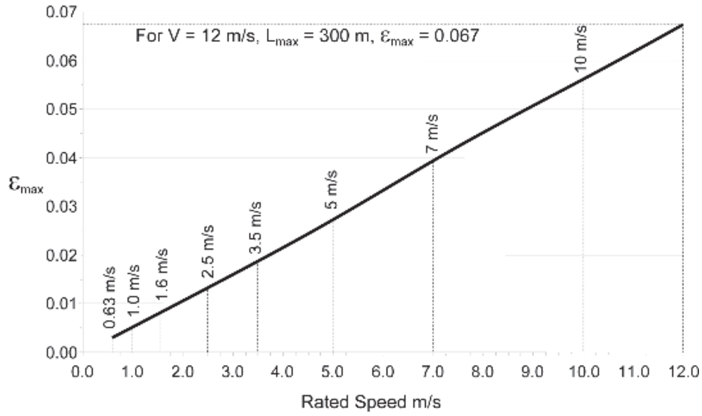

Figure 3: Values of ε_{max} relative to rated speed and travel

The result is shown in Figure 3, demonstrating that with this conservatively high estimate for factor of safety and maximum travel, we can say that at a rated speed of 12 mps,

$$\varepsilon_{max}\big|_{12m/s} = 0.067 \ll 1.0$$

allowing us to treat the elevator as quasi-stationary. Furthermore, given that experience shows that the problems usually arise near the upper limit of travel, in the area of interest in the hoistway the actual value of ε will be significantly lower than this maximum estimate. It turns out that at a rated speed of 12 mps, the value of ε at the slowdown point for the upper terminal floor is in the order of 0.052.

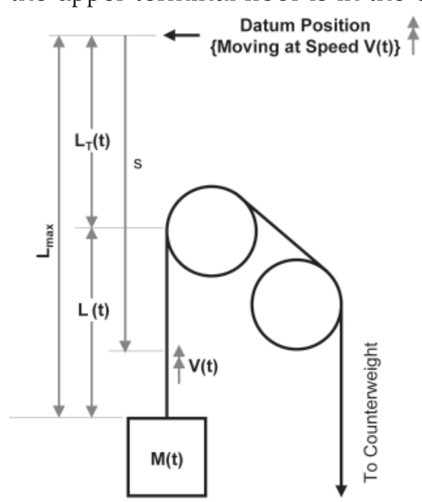

Figure 4: Dynamic model of the rope system

The Dynamic Model

Figure 4 shows a suitable dynamic model of the elevator system. In order to make the mathematics of the analysis more straightforward, the model employs a "moving frame of reference." In the model, all the distances are measured from a moving origin located at a fixed distance L_{max} above the car top, where L_{max} *(m)* represents the suspension-rope length when the car is at the lowest position. Thus, $L_T(t)$ represents the distance traveled by the car *(m); V(t)* is the elevator car speed (mps); *M(t)* is the car-side mass (kg) including the mass of any compensation ropes/chains, travelling cables, etc.; m_{sr} is the mass per unit length of the rope (kg/m); *T(t)* is the mean rope tension; *A* is the rope cross sectional area (9 mm^2); and *E* is the Young's modulus for the rope (N/mm^2).

Now we need some very clear thinking. If the ropes are still, i.e., not oscillating, then we can define a variable *s* to represent the position of any point along the length of the rope, relative to the (moving) datum position. In these conditions, we can say that the rope is "un-deformed," since it is only stretched by the suspended mass. At the extreme end, $s = L_{max}$ is the mean position of the point where the suspension meets the car top.

If a rope begins to oscillate, either vertically or laterally, it becomes dynamically deformed, i.e., a given point will be displaced from its quiescent position, *s* meters from the datum, by the (relatively) small amounts $\pm u$ (vertically), $\pm v$ (lateral in plane) and $\pm w$ (lateral out of plane).

Now let us examine what is happening if the system is in motion and oscillating. We will assume for the moment that the elevator machine and control system are "very stiff," i.e., any oscillations in the ropes are not propagated beyond the point where the ropes come into contact with the sheave *(the oscillations might themselves be initiated by oscillation propagated from the machine and/or control system, but since we are looking at the dynamics of the ropes themselves, we are assuming here that there is no "forcing function" coming from the sheave).* Since, in terms of motion through the hoistway, the system is "slowly varying," we can consider the motion of a point at position *s* along the ropes as if the elevator were actually stationary except for the oscillations. In these circumstances, the total energy in the system would be constant, and we can apply Hamilton's Principle[1], which states that over time, the time integral of the difference between the kinetic and potential energy in the system will be stationary. Note that in the context of the elasticity of the ropes, the potential energy of the system includes the strain energy in the ropes themselves. In mathematical terms,

$$\delta \int_{t_1}^{t_2} \{\text{K.E.} - (\text{P.E.} + \text{S.E.})\}\, dt = 0 \quad \text{............(Equation 10)}$$

where δ represents variation, K.E. is the system kinetic energy, P.E. is the gravitational potential energy and S.E. is the strain energy.

Be very clear about what Equation 10 means and what it doesn't. If the elevator car is accelerating downward, then the total kinetic energy of the system is increasing due to the acceleration, and the total potential energy is reducing, since motion is in the down direction (assuming that the car mass is greater than the counterweight mass). Conversely, if the car is slowing while traveling upward, total kinetic energy is reducing, while total potential energy is increasing.

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However, that is not the issue we are discussing here. Instead, we are taking a "snapshot" of the elevator car in motion at some point in the hoistway and looking at how the energy in the system at that instant is being transferred between the oscillatory motion of the ropes ($\dot{u}$, $\dot{v}$, $\dot{w}$, and $\ddot{u}$, $\ddot{v}$, $\ddot{w}$), the position of the point on the ropes (*u, v, w*) and the strain energy in the ropes. Of course, *u, v, w* and their time derivatives will vary depending on where we measure them along the rope, i.e., they will depend not only on time, but also on *s*, the location along the rope, so that at the moment when we take our snapshot, each is a function of both *s* and *t*. Again, in mathematical terms

$$u = u(s,t), \quad v = v(s,t), \quad w = w(s,t) \qquad \text{(Equation 11)}$$

To get any further with this analysis, we would need to go into the mathematics of classical mechanics. However, for the elevator engineer, it is the outcome of the analysis that is important, not the analysis itself. The outcome is a set of differential equations describing the oscillatory motion. We are not going to attempt any solution of these equations here, but simply present them to demonstrate that the three displacements *u, v,* and *w* are interdependent.

One set of three equations

$$m_{SR}\left(\frac{\partial^2 u}{\partial t^2} - \frac{d^2 L_T(t)}{dt^2}\right) - EA\frac{\partial}{\partial s}\left[\frac{\partial u}{\partial s} + \frac{1}{2}\left\{\left(\frac{\partial v}{\partial s}\right)^2 + \left(\frac{\partial w}{\partial s}\right)^2\right\}\right] - \frac{\partial T(t)}{\partial s} - m_{SR} g_n = 0$$

$$m_{SR}\frac{\partial^2 v}{\partial t^2} - EA\frac{\partial}{\partial s}\left\{\frac{\partial v}{\partial s}\left[\frac{\partial u}{\partial s} + \frac{1}{2}\left\{\left(\frac{\partial v}{\partial s}\right)^2 + \left(\frac{\partial w}{\partial s}\right)^2\right\}\right]\right\} - T(t)\frac{\partial^2 v}{\partial s^2} - \frac{\partial T(t)}{\partial s}\frac{\partial v}{\partial s} = 0$$

$$m_{SR}\frac{\partial^2 w}{\partial t^2} - EA\frac{\partial}{\partial s}\left\{\frac{\partial w}{\partial s}\left[\frac{\partial u}{\partial s} + \frac{1}{2}\left\{\left(\frac{\partial v}{\partial s}\right)^2 + \left(\frac{\partial w}{\partial s}\right)^2\right\}\right]\right\} - T(t)\frac{\partial^2 w}{\partial s^2} - \frac{\partial T(t)}{\partial s}\frac{\partial w}{\partial s} = 0$$

describes the motion of a point on the suspension ropes, while a fourth equation

$$\frac{M(t)}{n_{SR}}\left[\left.\frac{\partial^2 u}{\partial t^2}\right|_{s=L_{max}} - \frac{d^2 L_T(t)}{dt^2}\right] + \frac{1}{n_{SR}}\frac{dM(t)}{dt}\left[\left.\frac{\partial u}{\partial t}\right|_{s=L_{max}} - \frac{dL_T(t)}{dt}\right]$$
$$+ EA\left.\frac{\partial u}{\partial s}\right|_{s=L_{max}} + \left.T(t)\right|_{s=L_{max}} - \frac{M(t)}{n_{SR}} g_n = 0$$

describes the oscillating motion of the elevator car. Note that this final equation is evaluated at the position $s = L_{max}$, i.e., at the mean position where the suspension meets the car top. At this position, $v = w = 0$, since the ropes cannot move laterally where they are attached to the car, so the equation does not include any terms relating to *v* and *w* or their derivatives.

While this set of four equations is extremely complex, the point of interest for the elevator engineer is simply that all of the three motions *u, v* and *w* appear in each differential equation of the set describing the rope oscillations. This indicates that the three motions are coupled one to the other and will interact. Thus, a lateral oscillation of the ropes can generate a longitudinal oscillation and vice-versa, and any and all of the modes of oscillation will lead to oscillations of the elevator car.

It's fairly clear that at any instant, the mean tension at any point in the suspension rope will be

$$T(s) = \left\{\frac{M(t)}{n_{SR}} + m_{SR}\left(L_{max} - s\right)\right\} g_n \qquad \left[s \geq L(t)\right]$$

Note that since we are assuming that the ropes are oscillating, this is the mean tension. The actual instantaneous tension will depend on the amplitude and frequency of the oscillations.

Continued

From the set of differential equations above, the natural frequencies of the longitudinal oscillations can be determined from the equation

$$\omega_{un} = \gamma_n \sqrt{\frac{EA}{m_{SR}}} \quad n = 1, 2, 3, \ldots \qquad \text{(Equation 12)}$$

where γ_n are the solutions of

$$\gamma_n \tan\left(\gamma_n L(t)\right) - \frac{n_{SR} m_{SR}}{M(t)} = 0 \qquad \text{(Equation 13)}$$

and $L(t) = L_{max} - L_T(t)$ (Figure 4)

Thus, in this more detailed analysis of the rope oscillation, we find that, contrary to the simplistic prediction of Equation 1, there are possible (and likely) "harmonics" of the longitudinal natural frequency of oscillation of the ropes.

Based on the "slowly varying" criterion, we can use Equation 5 to account for lateral oscillations at any point in the hoistway:

$$\overline{\omega_{xn}} = \frac{n\pi}{L_{max} - L_T(t)} \sqrt{\frac{M(t) g_n}{n_{SR} m_{SR}} - \frac{\left[L_{max} - L_T(t)\right] g_n}{2}} \qquad \text{(Equation 14)}$$

As we did earlier, we can substitute in Equations 13 and 14 from the relation

$$\frac{M(t) g_n}{m_{SR} n_{SR}} = \frac{F_{min}}{S_f(t) m_{SR}} - L(t) g_n = \frac{F_{min}}{S_f(t) m_{SR}} - \left(L_{max} - L_T(t)\right) g_n$$

treating the variation of suspended mass over time as a variation in the safety factor, i.e.

$$\gamma_n \tan\left(\gamma_n L_T(t)\right) - \frac{S_f(t) m_{SR} g_n}{F_{min} - S_f(t)\left(L_{max} - L_T(t)\right) m_{SR} g_n} = 0$$

and $$\overline{\omega_{xn}} = \frac{n\pi}{L_{max} - L_T(t)} \sqrt{\frac{F_{min}}{S_f(t) m_{SR}} - \frac{\left\{L_{max} - L_T(t)\right\} g_n}{2}} \qquad \text{(Equation 15)}$$

thereby expressing the frequency equations in terms of the rope characteristics.

There is a number of possibilities for the excitation of oscillations in the ropes:

- The excitation may be generated from the machine and/or control system through:
  - Sheave/pulley eccentricity
  - Cyclic phenomena in a speed-reduction unit (e.g., number of starts on a worm shaft)
  - Electromagnetic phenomena in the motor (asymmetrical rotor windings in DC machines, spurious conducting paths in the rotor construction, e.g., uninsulated core bolts)
  - Frequency instability in the motor drive
- Eccentric roller guide shoes
- Impulsive input from one or more guide joints
- Guide misalignment
- It may occur that one or more of the longitudinal resonant frequencies predicted by Equation 12 coincides with a lateral frequency predicted by Equation 14.
- The building itself may have resonant frequencies coincident with one or more of the frequencies predicted by Equation 12 and/or Equation 14.

Example

Consider an elevator with the following parameters:

Car-side fixed mass	P	1600	kg
Rated load	Q	1250	kg
Reeving factor	r	2:1	
Suspension-rope length with car at lowest position	L_0	60	m
Number of suspension ropes	n_R	6	
Number of compensation ropes	n_{CR}	4	
Number of traveling cables	n_{TC}	3	
Suspension rope mass/m	m_R	1.2	kg/m
Compensation rope mass/m	m_{CR}	1.6	kg/m
Traveling cable mass/m	m_{Trav}	0.5	kg/m

Suppose that the rated speed is 3.5 mps and that the traction sheave has a diameter of 560 mm but is slightly eccentric, generating a longitudinal disturbance to the suspension ropes at a frequency of approximately 4 Hz when the elevator is running at rated speed. The first four calculated longitudinal natural frequencies of the suspension ropes (ω_{0u}, ω_{2u}, ω_{3u} and ω_{4u}) are shown in Figure 5 plotted against the suspension-rope length.

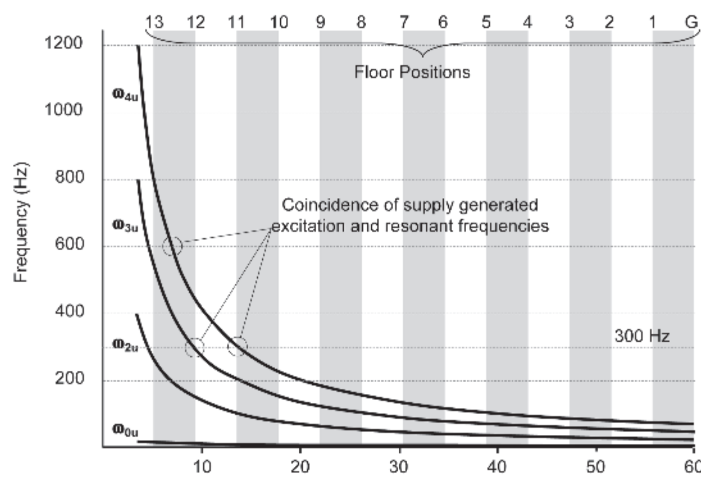

Figure 5: Longitudinal natural frequencies

Superimposed on the plot are shaded areas indicating the floor positions of the building. Each floor position is located at the boundary of a shaded area as shown. It is clear from the diagram that the fundamental longitudinal frequency (ω_{ou}) is quite low, as might be predicted from experience.

However, when we look at the second, third and fourth resonant frequencies, we see quite clearly how these increase as the elevator approaches the highest position. In a motor drive linked to mains frequency (e.g., a variable-voltage DC drive), it is likely that there will be a certain level of excitation generated by the drive at either 300 Hz or 600 Hz (based on a 50-Hz supply) or 360 Hz or 720 Hz (based on a 60-Hz supply). Clearly, at a number of locations between the 11th and 13th floors, any such excitation will coincide with one of the natural frequencies of the suspension and may well generate an associated vibration in the elevator car – a phenomenon of transient vibration as the elevator approaches the upper floors, which is well observed in practice, as we indicated in Figure 1.

If we now consider the lateral frequencies (Figure 6), superimposed on the diagram is the variation in the fundamental (lowest) longitudinal natural frequency (dotted line). The horizontal (double-dotted) lines show the location of the fundamental frequency and second harmonic frequency of the eccentric pulley.

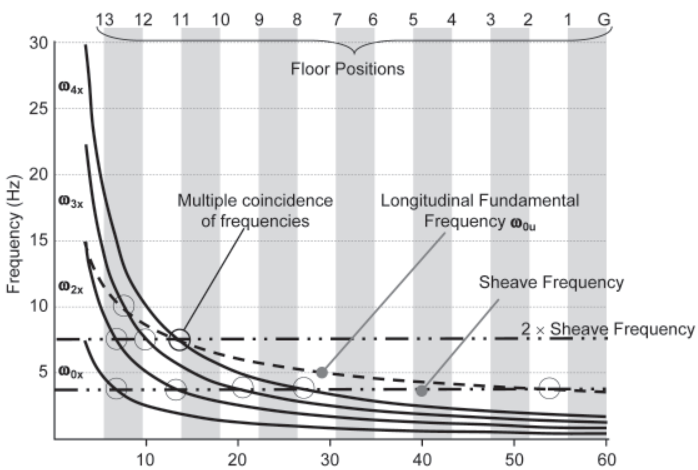

Figure 6: Lateral natural frequencies

The nine points where there is a coincidence between the pulley frequency and one of the natural frequencies are indicated by circles, starting with a coincidence located between the first and second floors where the longitudinal frequency coincides. This low frequency might be significant in that it could possibly excite a natural frequency of the compensation ropes, initiating rope sway beneath the car or oscillation of the compensator mass.

Of particular interest is the coincidence between the second harmonic of the sheave frequency, the longitudinal fundamental frequency and the fourth lateral frequency at approximately 7.5 Hz just above the 11th floor, with an additional coincidence between the second lateral frequency and the fundamental pulley frequency at around 4 Hz at almost the same position. There is a further coincidence of resonances as the elevator runs into the top floor. It could be predicted that this elevator might have some serious vibration problems, particularly around the 11th floor.

Summary

Transient vibrations at certain locations within the elevator hoistway are a well-recognized phenomenon. By considering the elevator suspension as a slowly varying system, the dynamic equations for lateral and longitudinal displacements can be established by the application of Hamilton's Principle, which suggests that the variation in the time integral of the difference between the kinetic energy and the potential energy of the system is stationary over time, i.e.,

$$\delta\int_{t_1}^{t_2}\{K.E.-(P.E.+S.E.)\}\,dt=0 \qquad \text{. . . . .(Equation 10)}$$

The set of nonlinear differential equations that results from the analysis clearly indicates that there is cross coupling between longitudinal and lateral oscillations in the ropes. Consequently, if, at some location in the hoistway, the longitudinal and lateral natural frequencies coincide, there is the likelihood that this cross coupling will excite a transient vibration in the car. The numerical example demonstrates that there is the potential for transient oscillations at several locations in the hoistway, particularly near the upper floors, where external forcing influences such as minor sheave eccentricity may coincide with one or more of the natural frequencies of the system.

While it is useful to explain the source of such transient vibrations and to be able to predict where and how they might arise, a practical engineer will question what palliative or curative measures can be taken to reduce or eliminate the effect. Experience demonstrates that this is far from a simple matter. The resonant frequencies are a function of the rope tension and rope length, which makes it quite difficult to move the resonance out of the way, since these parameters are fundamental to the elevator installation. Increasing the mass of the elevator would lower the resonant frequencies, but that would probably mean that the resonance simply moved to a location further up the hoistway. It would be fortuitous if it were possible to lower the resonant frequencies sufficiently to move the location of the resonance beyond the highest point in the travel.

The major contribution to the problem arises from the rope characteristics. As we noted at the beginning of the discussion, conventional steel wire-rope construction provides a suspension member with very little damping, i.e., once the vibration has been started, there is not much in the rope construction to absorb or dissipate the vibration energy. It is the absorption/dissipation of the vibration energy that is key to alleviating rope transmitted vibrations. New rope materials such as Kevlar® have better damping characteristics and should be less prone to the problem. However, these types of rope do not (yet) find universal application, and while they are becoming more common, extensive service experience such as is available with traditional steel wire ropes has not yet been built up.

Continued

Several authors have proposed a range of methods for damping out rope sway in compensation and suspension ropes.[1,3 & 6] Robertson[3], Barker[4] and Traktovenko[5] have patented mechanical methods to restrain the amplitude of rope oscillation at one or more points between the elevator car and the end of the hoistway. Salmon and Hiller[6] patented a hydraulic tie-down system for the compensator to minimize sway in the compensation ropes.

It is conceivable that an intelligent, active anchorage on the elevator car might be used to isolate the car from any oscillation in the suspension ropes, but such a system would be complex. Bearing in mind that the rope anchorage is fundamental to the integrity of the suspension, the safety implications of such an approach would also need considerable investigation and might have difficulty in the context of the essential safety requirements inherent in elevator safety codes.

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Phil Andrew joined Express Lift Co. in Northampton, the U.K., in 1978. Over the next 18 years, he held a range of positions with Express Lift. In 1996, he joined the elevator engineering group at the University of Northampton (then Nene College) as a Senior Lecturer, teaching the Lift and Escalator Industry Association Professional Diploma in Engineering (Lift and Escalator Technology). He led the team that developed the internationally recognized Northampton MSc in Lift Engineering, then the Foundation Degree in Lift Engineering. In 2003, he was promoted to Principal Lecturer and Divisional Leader for engineering. From 2000 until his retirement in 2004, Andrew served on the National Interest Review Committee for the ASME/ANSI A17 code committee and represented The University on Committee MHE/4 of the British Standards Institution responsible for lift safety standards within the U.K.

Dr. Stefan Kaczmarczyk joined the elevator engineering group at the University of Northampton (then the University College Northampton) as a senior lecturer in 2002. In 2003, he became the course leader for MSc Lift Engineering, then postgraduate field leader for Lift Engineering. He is currently Professor of Applied Mechanics at The University of Northampton. Dr. Kaczmarczyk has published over 70 journal and international conference papers in the area of vertical transportation and elevator engineering. He has been serving on the Institute of Physics Applied Mechanics Group Committee and initiated and developed a series of international conferences linking the U.K. and overseas-based academic and industrial research groups working in the area of mechanics of slender structures, with particular emphasis on applications in elevator engineering.

Learning-Reinforcement Questions

Use the below learning-reinforcement questions to study for the Continuing Education Assessment Exam available online at www.elevatorbooks.com or on page 118 of this issue.

- If an elevator is exhibiting transient vibration due to lateral rope vibration, what is the effect of increasing the safety factor of the ropes (i.e., reducing the load per rope)?
- Why does the value of the parameter ε determine whether the elevator may be considered "quasi stationary"?
- Why can we predict that horizontal rope oscillations may give rise to vertical oscillation and vice versa?
- Contrary to the simple model of a mass oscillating on the end of a spring, when a rope under tension is modeled using Hamilton's Principle, the model predicts multiple longitudinal resonant frequencies. Why should this be so?
- Why does resonant vibration depend only upon the elevator position, while the influence of elevator speed can be considered small?
- Given that any transient resonant vibration relates more to elevator position than to elevator speed, what is likely to be the most successful strategy for eliminating the problem in any particular case?
- When evaluating the parameter ε, we based the calculation on the lowest lateral resonant frequency, rather than the lowest longitudinal resonant frequency. Why?
- In evaluating whether an elevator may be considered "quasi stationary," we based our calculations on a rope factor of safety of 30. Given that safety codes permit a safety factor as low as 10 or 12, why would we base the calculation on such a high value?
- What do we mean by "transient vibration"?
- When looking at the lateral oscillations of the ropes, why is the simple expression

$$\omega_{x0} = \frac{\pi}{L(t)}\sqrt{\frac{\text{Rope tension}}{\text{Rope mass/meter}}}$$

not adequate for predicting the onset of transient vibration?

ELEVATOR WORLD Continuing Education Assessment Examination Questions

Instructions:

- Read the article **"Rope Dynamics"** (page 45) and study the learning-reinforcement questions at the end of the article.
- To receive **two hours (0.2 CEUs)** of continuing-education credit, answer the assessment examination questions found below online at www.elevatorbooks.com or fill out the ELEVATOR WORLD Continuing Education Reporting Form found overleaf and submit by mail with payment.
- Approved for Continuing Education by **NAEC for CET® and NAESA International and QEI Services, Inc. for QEI.**

1. Which of the following statements relating to lateral oscillations of a suspension rope is *incorrect*?
   a. In calculating lateral oscillation frequencies, we must account for the mass of the suspension rope, as well as the mass of the elevator car.
   b. The vertical orientation of the suspension ropes does not affect the oscillation frequency.
   c. The mass of the suspension ropes increases the oscillation frequency to a slightly higher value than would be predicted by the simple theory of a stretched string.
   d. As the elevator travels up the hoistway and the suspension ropes become shorter, the lateral natural frequencies of the ropes increase.
2. Consider the following two statements regarding the "quasi-stationary" nature of an elevator:
   i. If the ratio

$$\frac{\text{Elevator rated speed}}{\text{Lowest resonant frequency x Maximum rope length}}$$

   is much smaller than unity, then the elevator may be considered "quasi stationary."
   ii. Because an elevator may be considered "quasi-stationary," we can apply Hamilton's Principle to determine the differential equations of motion of any point along the ropes between the car top and the traction sheave.
   a. Statement i is true, and statement ii is false.
   b. Both statements are false.
   c. Both statements are true.
   d. Statement i is false, and statement ii is true.
3. Transient resonant vibration is most likely:
   a. Near the bottom of the hoistway.
   b. In the middle of the hoistway.
   c. Equally likely at any point in the hoistway.
   d. Near the top of the hoistway.
4. Which of the following statements is *incorrect*?
   a. Transient vibration may occur if one of the longitudinal or lateral natural frequencies of the rope system coincides with an external excitation (e.g., poorly aligned guide rail joints, imperfections in the gearbox, eccentric reeving pulleys, etc.).
   b. Transient vibration will be more likely if there is coincidence between any of the lateral resonant frequencies and one of the longitudinal resonant frequencies.
   c. Vibration is transient, because the stiffness of the ropes damps out the vibrations after a short time.
   d. Transient vibration depends on position in the hoistway, not upon the rated speed of the elevator.
5. The statements below relate to Hamilton's Principle and the vibration of the elevator car:
   i. The expression

$$\delta\int_{t_1}^{t_2}\left(\text{Kinetic Energy} - \left(\text{Potential Energy} + \text{Strain Energy}\right)\right)dt = 0$$

   defining Hamilton's principle is concerned with the oscillatory motion of each small element of the rope, not with the overall energy of the system.
   ii. Because the differential equation

$$\frac{M(t)}{n_{SR}}\left[\left.\frac{\partial^2 u}{\partial t^2}\right|_{s=L_{max}} - \frac{d^2L_T(t)}{dt^2}\right] + \frac{1}{n_{SR}}\frac{dM(t)}{dt}\left[\left.\frac{\partial u}{\partial t}\right|_{s=L_{max}} - \frac{dL_T(t)}{dt}\right] + EA\left.\frac{\partial u}{\partial s}\right|_{s=L_{max}} + T(t)\big|_{s=L_{max}} - \frac{M(t)}{n_{SR}}g_n = 0$$

   describing the oscillation of the elevator car does not contain the parameters *v*(*s*,*t*) and *w*(*s*,*t*) defining *lateral* motion of the ropes, vibration in the elevator car is independent of any lateral vibration of the ropes.
   a. Statement i is true, and statement ii is false.
   b. Both statements are false.
   c. Both statements are true.
   d. Statement i is false, and statement ii is true.
6. We have used the symbols *u*, *v* and *w* to indicate oscillatory displacement in each of the three orthogonal planes: longitudinal, lateral in-plane and lateral out-of-plane. With regard to the *u*, this indicates:
   a. Vertical motion of a point on the ropes, including the speed of travel of the elevator car.
   b. The longitudinal displacement of a point on the ropes from its quiescent position.
   c. Vertical position of a point on the ropes as the elevator travels through the hoistway.
   d. The longitudinal frequency of oscillation of a point on the ropes about its quiescent position.
7. If the lateral oscillation of the ropes is in a direction *x* at an angle θ to the plane of the guides, then if *v* indicates displacement in the plane of the guides, and *w* indicates displacement orthogonal to the plane of the guides:

a. $v = x\sin\theta$, $w = v\cos\theta$.
b. $v = x\sin\theta$, $w = x\cos\theta$.
c. $w = x\cos\theta$, $v = w\sin\theta$.
d. $v = x\cos\theta$, $w = x\sin\theta$.

8. In assessing the lateral oscillation frequencies of the ropes, we must take account of:
   a. The rope tension at the car top (i.e., excluding the weight of the ropes themselves).
   b. The rope tension at the sheave (i.e., including the weight of the ropes themselves).
   c. The mean rope tension, including 50% of the weight of the ropes themselves.
   d. The mean rope tension, including 75% of the weight of the ropes themselves.
9. In assessing whether an elevator is a "slowly varying" system, we must evaluate the dimensionless parameter ε:
   a. With the maximum factor of safety (i.e., minimum load on the ropes) and at the lowest point in the travel (i.e., maximum rope length).
   b. With the minimum factor of safety (i.e., maximum load on the ropes) and at the lowest point in the travel (i.e., maximum rope length).
   c. With the maximum factor of safety (i.e., minimum load on the ropes) and at the highest point in the travel, with the elevator traveling at rated speed (i.e., at the slowdown point for the highest floor).
   d. With the minimum factor of safety (i.e., maximum load on the ropes) and at the highest point in the travel with the elevator traveling at rated speed (i.e., at the slowdown point for the highest floor).
10. A number of external influences may lead to rope vibration or oscillation during travel. Some of these may be:
   - Sheave/pulley eccentricity
   - Cyclic phenomena in a speed-reduction unit (e.g., number of starts on a worm shaft)
   - Electromagnetic phenomena in the motor (asymmetrical rotor windings in DC machines, spurious conducting paths in the rotor construction [e.g., uninsulated core bolts])
   - Frequency instability in the motor drive
   - Eccentric roller-guide shoes
   - Poorly aligned guide joints
   - Guide misalignment
   - It may occur that one or more of the longitudinal resonant frequencies coincides with a lateral frequency.
   - The building itself may have resonant frequencies coincident with one or more of the roping-system frequencies.

   Which of the following statements is *correct*?
   a. The building itself cannot influence the suspension system.
   b. Any or all of the above may be the cause.
   c. Guide misalignment will not lead to rope oscillation.
   d. Poorly aligned guide joints will only give a "jolt" and not lead to rope oscillation.
11. Vibration generated from the mains supply (e.g., through the drive system for the elevator motor) is more likely to lead to longitudinal rather then lateral vibrations. Which of the following is *not* a likely explanation of this phenomenon?
   a. The excitation is transmitted via the traction sheave, and the traction sheave will not vibrate laterally.
   b. The longitudinal resonant frequencies are higher than the lateral resonant frequencies.
   c. The lateral resonant frequencies generally fall in a range below the frequency of the mains supply.
   d. In the upper part of the hoistway, the longitudinal resonant frequencies may coincide with the harmonics of the mains frequency.
12. i. Sheave/pulley eccentricity will always generate oscillations in the suspension ropes through the whole length of the hoistway, simply because the point of contact between the pulley and ropes is oscillating.
   ii. The oscillation frequency generated by an eccentric roller-guide shoe may fall in the range of the lateral resonant frequencies of the suspension ropes at some point in the hoistway.
   a. Statement i is true, and statement ii is false.
   b. Both statements i and ii are false.
   c. Both statements i and ii are true.
   d. Statement i is false, and statement ii is true.
13. Our analysis of rope vibration is based on the premise that the system parameters are "slowly varying." Given a slowly varying system, which of the following statements is *incorrect*?
   a. In our dynamic model of the elevator system, the datum point from which we make our distance measurements moves at the same speed as the elevator car.
   b. In analyzing rope vibration, the rope speed (not the elevator speed) is important.
   c. The lowest resonant frequencies of the compensation ropes/chains occur when the elevator is near the highest floor.
   d. The lowest resonant frequencies of the suspension ropes occur when the elevator is near the lowest floor.
14. i. If an elevator is suffering from transient vibration, changing the rope characteristics (e.g., using more, smaller ropes or replacing the existing suspension with larger ropes) is likely to produce a significant improvement in performance.
   ii. Because they are softer and lighter than conventional steel-wire ropes, ropes made from new materials such as Kevlar® are likely to be more prone to rope vibration.

Continued

a. Statement i is true, and statement ii is false.
b. Both statements i and ii are false.
c. Both statements i and ii are true.
d. Statement i is false, and statement ii is true.

15. Our analysis of rope vibration is based on rope factor of safety and rope minimum breaking load, rather than elevator mass and overall rope mass:
a. Because minimum rope factor of safety is a fundamental parameter defined by safety codes.
b. Because the minimum breaking load of the rope is an essential parameter of the design.
c. In order to relate the characteristics of the vibration to the characteristics of the rope (and not to the masses) in a particular installation.
d. In order to emphasize the relationship between actual rope loading and the expected service life of the ropes.

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